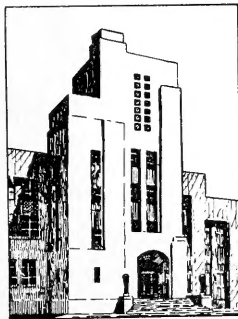# NAVY DEPARTMENT

THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

THE CONFIGURATION AND TENSION OF A LIGHT FLEXIBLE
CABLE IN A UNIFORM STREAM

RESEARCH AND DEVELOPMENT REPORT

March 1956

Appendix 1
Report 717

FOREWORD

This appendix should be included in TMB Report 717 for the purpose of clarifying the definition of symbols. It is an unclassified reprint of Appendix 1, TMB Report C-122, June 1948.

THE CONFIGURATION AND TENSION OF A LIGHT FLEXIBLE
CABLE IN A UNIFORM STREAM

The problem of determining the shape and tension of a light flexible round cable in a uniform stream was treated in a previous report (4). The same problem is treated here, but a modified law due to Reber (5) for the hydrodynamic force acting on an element of cable is assumed. The weight of the cable is neglected. The quadratures that arise in the solution of the differential equations that describe the cable configuration and tension thereby become explicitly integrable, so that the configuration and tension can be expressed by functions that have been tabulated. Consequently the results are easy to apply to problems involving cables whose parameters differ from those of a round cable, provided, of course, that the same law of force holds. In particular, the method can be applied to problems that involve faired cables.

The hydrodynamic force acting on an element of cable is assumed to lie in the plane containing the direction of the stream and the direction of the element of cable and is assumed to depend only on the angle between these directions. Consequently the entire configuration of the cable lies in a plane.

The following law of force is assumed; see Figure 16. The hydrodynamic force acting on an element of cable is considered to consist of two parts:

1. A profile drag that acts normal to the cable and varies as the square of the sine of the angle that the element makes with the stream.

2. A frictional drag that acts in line with the stream and has a magnitude that is independent of the angle that the element makes with the stream.

Choose a point O on the cable as origin of a rectangular coordinate system; see Figure 17. In general, the angle to the stream and the tension in the cable at this point will be known. The X-direction is taken as the direction of motion of the cable. The Y-direction is taken as positive toward that side of the x-axis which is reached by proceeding along the cable upstream from the origin, O. An arbitrarily chosen point P on the cable is thus assigned coordinates x, y.

When the cable forms a loop and the origin of the coordinate system is chosen as the point where the cable is normal to the stream, any movement
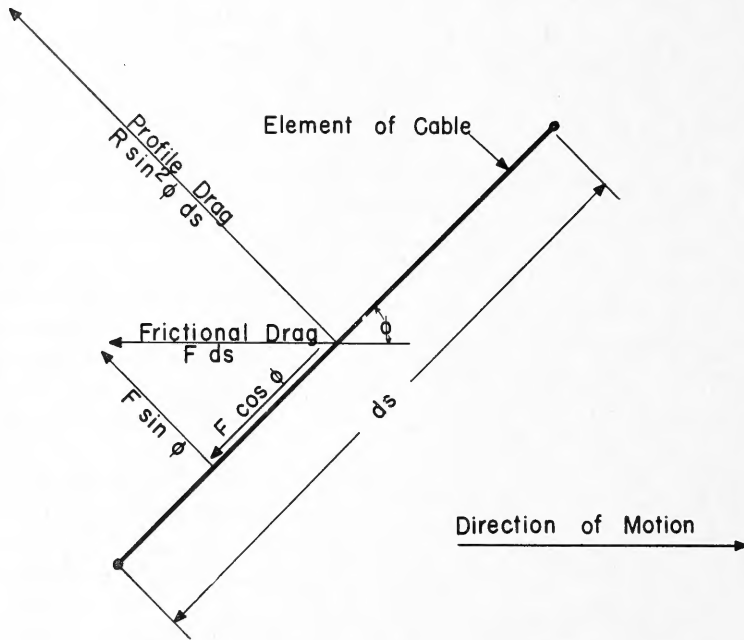
Figure 16 - Resolution of Hydrodynamic Force Acting
upon an Element of Cable

along the cable from the origin O would lead upstream. In this case the positive direction of Y may be chosen arbitrarily.

The distance along the cable from the origin O to the arbitrarily chosen point P on the cable is denoted by s. Distances along the cable to points that are reached by proceeding from the origin O in the direction of increasing Y are measured in a positive sense, whereas distances along the cable from the origin O to points that are reached by proceeding from the origin O in the direction of decreasing Y are measured in a negative sense.

The angle of the cable to the stream at the arbitrarily chosen point P is denoted by ϕ and is measured from the positive X-direction to the positive direction of the tangent at the point P. The positive direction of the tangent to the cable at P is determined by the direction of increasing s. The angle of the cable to the stream at the origin O is denoted by ϕ_0.

The tension in the cable at the arbitrarily chosen point P and at the origin O are denoted by T and T_0 respectively.
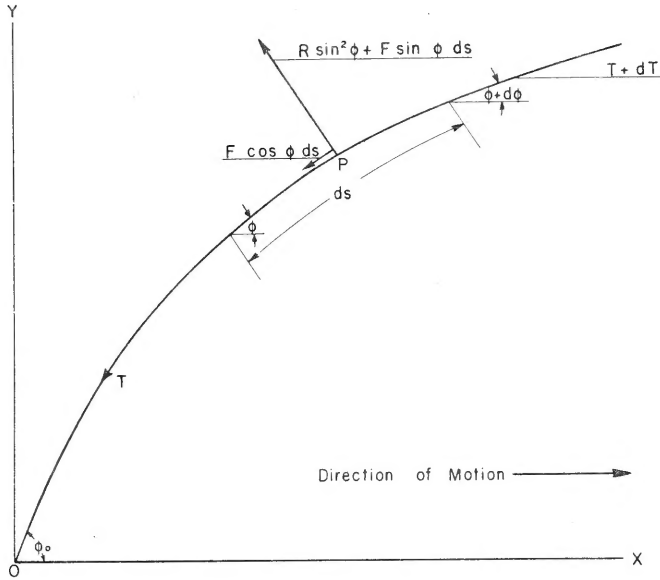
Figure 17 - Coordinate System

Figure 17 shows the forces acting on a cable element at a point P. The element ds of the cable at the point P is in equilibrium under the action of the hydrodynamic forces and the tensions in the cable, $T + dT$ and T, which act respectively at the forward and after ends of the element of the cable. By resolving forces, first tangent to and second normal to the cable, we arrive at the following differential equations:

$$dT = F \cos \phi \, ds \qquad\qquad [1]$$

$$T \, d\phi = (-R \sin^2 \phi - F \sin \phi) \, ds \qquad\qquad [2]$$

where F is the drag per unit length of the faired cable when the cable is parallel to the stream, and

R is the form drag per unit length of the faired cable when the cable is normal to the stream; and 180 deg $> \phi > 0$.

From the geometry shown in Figure 17

$$dx = \cos \phi \, ds \qquad\qquad [3]$$

$$dy = \sin\phi \, ds \tag{4}$$

Equations [1], [2], [3], and [4] taken together completely define the tension and configuration of the cable when the end conditions T_0 and ϕ_0 are known.

Equation [1] is immediately integrable to

$$T = T_0 + Fx \tag{5}$$

Eliminating ds from Equations [1] and [2] and substituting f for $\frac{F}{R}$ we obtain

$$\frac{dT}{T} = \frac{-f\cos\phi \, d\phi}{\sin^2\phi + f\sin\phi} = \frac{-f\csc\phi\cot\phi \, d\phi}{1 + f\csc\phi} \tag{6}$$

which integrates to give

$$T = \frac{1 + f\csc\phi}{1 + f\csc\phi_0} \, T_0 \tag{7}$$

Substituting this value of T into Equation [2]

$$\frac{R \, ds}{T_0} = \frac{-(1 + f\csc\phi) \, d\phi}{(1 + f\csc\phi_0)(\sin^2\phi + f\sin\phi)} = \frac{-\csc^2\phi \, d\phi}{1 + f\csc\phi_0} \tag{8}$$

This integrates to give

$$\frac{R s}{T_0} = \frac{\cot\phi - \cot\phi_0}{1 + f\csc\phi_0} \tag{9}$$

From Equations [8] and [3] we obtain

$$\frac{R \, dx}{T_0} = \frac{-\csc\phi \cot\phi \, d\phi}{1 + f\csc\phi_0} \tag{10}$$

which gives upon integration

$$\frac{Rx}{T_0} = \frac{\csc\phi - \csc\phi_0}{1 + f\csc\phi_0} \tag{11}$$

From Equations [8] and [4] we obtain

$$\frac{R \, dy}{T_0} = \frac{-\csc\phi \, d\phi}{1 + f\csc\phi_0} \tag{12}$$

which gives upon integration

$$\frac{Ry}{T_0} = \frac{\ln\dfrac{\cot\phi/2}{\cot\phi_0/2}}{1 + f\csc\phi_0} \tag{13}$$

A useful form of this relation is found by solving for functions of ϕ. Thus:

$$\cot \frac{\phi}{2} = e^{\frac{Ry}{T_0}(1 + f \csc \phi_0) + \ln \cot \frac{\phi_0}{2}}$$ [14]

and applying trigonometric identities gives

$$\csc \phi = \frac{\cot \frac{\phi}{2} + \tan \frac{\phi}{2}}{2} = \cosh\left[\frac{Ry}{T_0}(1 + f \csc \phi_0) + \ln \cot \frac{\phi_0}{2}\right]$$ [15]

$$\cot \phi = \frac{\cot \frac{\phi}{2} - \tan \frac{\phi}{2}}{2} = \sinh\left[\frac{Ry}{T_0}(1 + f \csc \phi_0) + \ln \cot \frac{\phi_0}{2}\right]$$ [16]

We may now easily obtain formulas for T, s, and x in terms of y and the end conditions T_0 and ϕ_0 at the origin O. From Equations [7] and [15]

$$T = \frac{T_0\left(1 + f \cosh\left[\frac{Ry}{T_0}(1 + f \csc \phi) + \ln \cot \frac{\phi_0}{2}\right]\right)}{1 + f \csc \phi_0}$$ [17]

From Equations [9] and [16]

$$s = \frac{T_0}{R}\left[\frac{\sinh\left\{\frac{Ry}{T_0}(1 + f \csc \phi_0) + \ln \cot \frac{\phi_0}{2}\right\} - \cot \phi_0}{1 + f \csc \phi_0}\right]$$ [18]

and from Equations [11] and [15]

$$x = \frac{T_0}{R}\left[\frac{\cosh\left\{\frac{Ry}{T_0}(1 + f \csc \phi_0) + \ln \cot \frac{\phi_0}{2}\right\} - \csc \phi_0}{1 + f \csc \phi_0}\right]$$ [19]

We may also express T, x, and y in terms of s and the end conditions, by solving Equation [9] for functions of ϕ. These relations are:

$$T = \frac{T_0\left[1 + f\left\{1 + \left[(1 + f \csc \phi_0)\frac{Rs}{T_0} + \cot \phi_0\right]^2\right\}^{\frac{1}{2}}\right]}{1 + f \csc \phi_0}$$ [20]

$$x = \frac{T_0}{R}\left[\frac{\left\{1 + \left[(1 + f \csc \phi_0)\frac{Rs}{T_0} + \cot \phi_0\right]^2\right\}^{\frac{1}{2}} - \csc \phi_0}{1 + f \csc \phi_0}\right]$$ [21]

$$y = \frac{T_0}{R(1 + f \csc \phi_0)} \ln\left[\frac{\left\{1 + \left[(1 + f \csc \phi_0)\frac{Rs}{T_0} + \cot \phi_0\right]^2\right\}^{\frac{1}{2}} + \left\{(1 + f \csc \phi_0)\frac{Rs}{T_0} + \cot \phi_0\right\}}{\cot \frac{\phi_0}{2}}\right] \quad [22]$$

Similarly solving Equation [11] for functions of ϕ we obtain the following formulas for T, s, and y.

$$T = T_0 \frac{1 + f\left\{(1 + f \csc \phi_0)\frac{Rx}{T_0} + \csc \phi_0\right\}}{1 + f \csc \phi_0} = T_0\left[1 + f \frac{Rx}{T_0}\right] \quad [23]$$

$$s = \frac{T_0}{R}\left[\frac{\left\{\left[(1 + f \csc \phi_0)\frac{Rx}{T_0} + \csc \phi_0\right]^2 - 1\right\}^{\frac{1}{2}} - \cot \phi_0}{1 + f \csc \phi_0}\right] \quad [24]$$

$$y = \frac{T_0}{R(1 + f \csc \phi_0)} \ln\left[\frac{\left\{\left[(1 + f \csc \phi_0)\frac{Rx}{T_0} + \csc \phi_0\right]^2 - 1\right\}^{\frac{1}{2}} + (1 + f \csc \phi_0)\frac{Rx}{T_0} + \csc \phi_0}{\cot \frac{\phi_0}{2}}\right] \quad [25]$$

Thus when the cable parameters F and R and complete conditions are given at any one point, complete conditions can be found at once for any other point at which one value is known. Usually these points are the downstream and upstream ends of the cable respectively.

Greater difficulty is presented by problems which do not specify all the cable parameters or complete conditions at any one point. These problems can be treated by the methods described in Reference (4).

If we define

$$\tau = 1 + f \csc \phi \quad [26]$$

$$\sigma = \cot \phi \quad [27]$$

$$\xi = \csc \phi - 1 \quad [28]$$

$$\eta = \ln \cot \frac{\phi}{2} \quad [29]$$

and let τ_0, σ_0, ξ_0, and η_0 be the values of these functions for $\phi = \phi_0$ then from Equations [7], [9], [11], and [13]

$$\frac{T}{T_0} = \frac{\tau}{\tau_0} \quad [30]$$

$$\frac{Rs}{T_0} = \frac{\sigma - \sigma_0}{\tau_0} \qquad [31]$$

$$\frac{Rx}{T_0} = \frac{\xi - \xi_0}{\tau_0} \qquad [32]$$

$$\frac{Ry}{T_0} = \frac{\eta - \eta_0}{\tau_0} \qquad [33]$$

and it is seen that these functions take the place of the tabulated functions of Reference (4).

The functions σ, ξ, and η represent the cable configuration reduced to nondimensional form. It is interesting to note that the nondimensional configuration as given by Equations [27], [28], and [29] is that of a catenary and is independent of the frictional drag, F.